
10
SIMPLIFYING AND SOLVING

CHAPTER 10 Simplifying and Solving

Since the beginning of this course, you have studied several different types of equations and have developed successful methods to solve them. For example, you have learned how to solve linear equations, systems of linear equations, and quadratic equations.

In Chapter 10, you will **extend** your solving skills to include other types of equations, including equations with square roots, absolute values, and messy fractions.

Another focus of this chapter is on learning how to simplify algebraic fractions (called "rational expressions") and expressions with exponents. By using the special properties of the number 1 and the meaning of exponents, you will be able to simplify large, complicated expressions.

Guiding Questions

Think about these questions throughout this chapter:

How can I rewrite it?

How can I solve it?

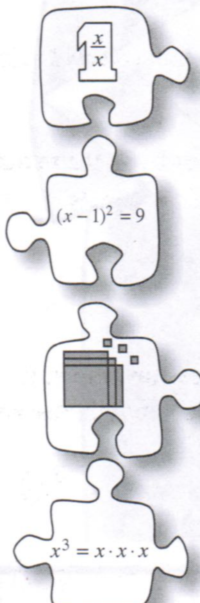
Is there another method?

What is special about the number 1?

In this chapter, you will learn how to:

- Simplify expressions involving exponents and fractions.
- Solve quadratic equations by completing the square.
- Use multiple methods to solve new types of equations and inequalities, such as those with square roots, rational expressions, and absolute values.

Chapter Outline



Section 10.1 In this section, you will study the properties of the number 1 and use them to simplify rational expressions and solve equations with fractions.

Section 10.2 Using the skills you learned in Section 10.1, you will develop new ways to solve unfamiliar, complicated equations involving square roots and absolute values.

Section 10.3 In this section, you will learn how to rewrite quadratics in perfect square form using a process called "completing the square."

Section 10.4 At the end of this chapter, you will use the meaning of an exponent to develop strategies to simplify exponential expressions.

10.1.1 How can I simplify?

Simplifying Expressions

In Chapter 8, you used the special qualities of the number zero to develop a powerful way to solve factorable quadratics. In Section 10.1, you will focus on another important number: the number 1. What is special about 1? What can you do with the number 1 that you cannot do with any other number? You will use your understanding of the number 1 to simplify algebraic fractions, which are also known as **rational expressions**.

10-1. What do you know about the number 1? Brainstorm with your team and be ready to report your ideas to the class. Create examples to help show what you mean.

10-2. Mr. Wonder claims that anything divided by itself equals 1 (as long as you do not divide by zero). For example, he says that $\frac{16x}{16x} = 1$ if x is not zero.

a. Is Mr. Wonder correct?

b. Why can't x be zero?

c. Next he considers $\frac{x-3}{x-3}$. Does this equal 1? What value of x must be excluded in this fraction?

d. Create your own rational expression (algebraic fraction) that equals 1. **Justify** that it equals 1.

e. Mr. Wonder also says that when you multiply any number by 1, the number stays the same. For example, he says that the product below equals $\frac{x}{y}$. Is he correct?

$$\frac{z}{z} \cdot \frac{x}{y} = \frac{x}{y}$$

10-3. Use what you know about the number 1 to simplify each expression below, if possible. State any values of the variables that would make the denominator zero.

a. $\frac{x^2}{x^2}$

b. $\frac{x}{x} \cdot \frac{x}{x} \cdot \frac{x}{3}$

c. $\frac{x-2}{x-2} \cdot \frac{x+5}{x-1}$

d. $\frac{9}{x} \cdot \frac{x}{9}$

e. $\frac{h \cdot h \cdot k}{h}$

f. $\frac{(2m-5)(m+6)}{(m+6)(3m+1)}$

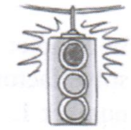
g. $\frac{6(n-2)^2}{3(n-2)}$

h. $\frac{3-2x}{(4x-1)(3-2x)}$

10-4. Mr. Wonder now tries to simplify $\frac{4x}{x}$ and $\frac{4+x}{x}$.

a. Mr. Wonder thinks that since $\frac{x}{x}=1$, then $\frac{4x}{x}=4$. Is he correct? Substitute three values of x to **justify** your answer.

b. He also wonders if $\frac{4+x}{x}=5$. Is this simplification correct? Substitute three values of x to **justify** your answer. Remember that $\frac{4+x}{x}$ is the same as $(4+x)\div x$.



c. Compare the results of parts (a) and (b). When can a rational expression be simplified in this manner?

d. Which of the following expressions below is simplified correctly? Explain how you know.

i. $\frac{x^2+x+3}{x+3}=x^2$ *ii.* $\frac{(x+2)(x+3)}{x+3}=x+2$

10-5. In problem 10-4, you may have noticed that the numerator and denominator of an algebraic fraction must both be written as a product before any terms create a 1. Examine the expressions below. Factor the numerator and denominator of each fraction, if necessary. That is, rewrite each one as a product. Then look for "ones" and simplify. For each expression, assume the denominator is not zero.

a. $\frac{x^2+6x+9}{x^2-9}$ b. $\frac{2x^2-x-10}{3x^2+7x+2}$ c. $\frac{28x^2-x-15}{28x^2-x-15}$ d. $\frac{x^2+4x}{2x+8}$

10-6. In your Learning Log, explain how to simplify rational expressions such as those in problem 10-5. Be sure to include an example. Title this entry "Simplifying Rational Expressions" and include today's date.

MATH NOTES

LOOKING DEEPER

Multiplicative Identity Property

When any number is multiplied by 1, its value stays the same.

For example:

$142\cdot 1=142$ $1\cdot k^2=k^2$ $\frac{4}{4}\cdot\frac{2}{3}=\frac{2}{3}$

Review & Preview

10-7. How many solutions does each equation below have?

a. $4x + 3 = 3x + 3$

b. $3(x - 4) - x = 5 + 2x$

c. $(5x - 2)(x + 4) = 0$

d. $x^2 - 4x + 4 = 0$

10-8. While David was solving the equation $100x + 300 = 500$, he wondered if he could first change the equation to $x + 3 = 5$. What do you think?

a. Solve both equations and verify that they have the same solution.

b. What did you do to the equation $100x + 300 = 500$ to change it into $x + 3 = 5$?

10-9. Solve each of the following inequalities for the given variable. Represent your solutions on a number line.

a. $5 + 3x < 5$

b. $-3x \geq 8 - x$

10-10. For each rational expression below, state any values of the variables that would make the denominator zero. Then complete each part.

a. Use the fact that $(x + 4)^2 = (x + 4)(x + 4)$ to rewrite $\frac{(x+4)^2}{(x+4)(x-2)}$. Then look for "ones" and simplify.

b. Use the strategy you used in part (a) to simplify the expression $\frac{8(x+2)^3(x-3)^3}{4(x+2)^2(x-3)^5}$.

10-11. In Lesson 10.1.2 you will focus on multiplying and dividing rational expressions. Recall what you learned about multiplying and dividing fractions in a previous course as you answer the questions below. To help you, the following examples have been provided.

$$\frac{9}{16} \cdot \frac{4}{6} = \frac{36}{96} = \frac{3}{8}$$

$$\frac{5}{6} \div \frac{20}{12} = \frac{5}{6} \cdot \frac{12}{20} = \frac{60}{120} = \frac{1}{2}$$

a. Without a calculator, multiply $\frac{2}{3} \cdot \frac{9}{14}$ and reduce the result. Then use a calculator to check your answer. Describe your method for multiplying fractions.

b. Without a calculator, divide $\frac{3}{5} \div \frac{12}{25}$ and reduce the result. Then use a calculator to check your answer. Describe your method for dividing fractions.

10-12. **Multiple Choice:** Which of the points below is a solution to $y < |x - 3|$?

a. (2, 1)

b. (–4, 5)

c. (–2, 8)

d. (0, 3)