10.1.2 How can I rewrite it?

Multiplying and Dividing Rational Expressions

In a previous course you learned how to multiply and divide fractions. But what if the fractions have variables in them? (That is, what if they are rational expressions?) Is the process the same? Today you will learn how to multiply and divide rational expressions and will continue to practice simplifying rational expressions.

10-13. Review what you learned yesterday as you simplify the rational expression at right. What are the excluded values of x? (That is, what values can x not be?)

$$\frac{3x^2+11x-4}{2x^2+11x+12}$$

10-14. With your team, review your responses to homework problem 10-11. Verify that everyone obtained the same answers and be prepared to share with the class how you multiplied and divided the fractions below.

$\frac{2}{3} \cdot \frac{9}{14}$ $\qquad$ $\frac{3}{5} \div \frac{12}{25}$

10-15. Use your understanding of multiplying and dividing fractions to rewrite the expressions below. Then look for "ones" and simplify. For each rational expression, also state any values of the variables that would make the denominator zero.

a. $\frac{4x+3}{x-5} \cdot \frac{x-5}{x+3}$ $\qquad$ b. $\frac{x+2}{9x-1} \div \frac{2x+1}{9x-1}$ $\qquad$ c. $\frac{2m+3}{3m-2} \cdot \frac{7+4m}{3+2m}$

d. $\frac{(y-2)^3}{3y} \cdot \frac{y+5}{(y+2)(y-2)}$ $\qquad$ e. $\frac{15x^3}{3y} \div \frac{10x^2y}{4y^2}$ $\qquad$ f. $\frac{(5x-2)(3x+1)}{(2x-3)^2} \div \frac{(5x-2)(x-4)}{(x-4)(2x-3)}$

10-16. PUTTING IT ALL TOGETHER

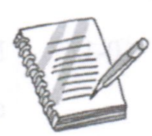
Multiply or divide the expressions below. Leave your answers as simplified as possible. For each rational expression, assume the denominator is not zero.

a. $\frac{20}{22} \cdot \frac{14}{35}$ $\qquad$ b. $\frac{12}{40} \div \frac{15}{6}$

c. $\frac{5x-15}{3x^2+10x-8} \div \frac{x^2+x-12}{3x^2-8x+4}$ $\qquad$ d. $\frac{12x-18}{x^2-2x-15} \cdot \frac{x^2-x-12}{3x^2-9x-12}$

e. $\frac{5x^2+34x-7}{10x} \cdot \frac{5x}{x^2+4x-21}$ $\qquad$ f. $\frac{2x^2+x-10}{x^2+2x-8} \div \frac{4x^2+20x+25}{x+4}$

10-17. In your Learning Log, explain how to multiply and divide rational expressions. Include an example of each. Title this entry "Multiplying and Dividing Rational Expressions" and include today's date.



MATH NOTES

METHODS AND MEANINGS

Rewriting Rational Expressions

To simplify a rational expression, both the numerator and denominator must be written in factored form. Then look for factors that make 1 and simplify. Study Examples 1 and 2 below.

Example 1: $\frac{x^2+5x+4}{x^2+x-12} = \frac{(x+4)(x+1)}{(x+4)(x-3)} = 1 \cdot \frac{x+1}{x-3} = \frac{x+1}{x-3}$ for $x \neq -4$ or 3

Example 2: $\frac{2x-7}{2x^2+3x-35} = \frac{(2x-7)(1)}{(2x-7)(x+5)} = 1 \cdot \frac{1}{x+5} = \frac{1}{x+5}$ for $x \neq -5$ or $\frac{7}{2}$

Just as you can multiply and divide fractions, you can multiply and divide rational expressions.

Example 3: Multiply $\frac{x^2+6x}{(x+6)^2} \cdot \frac{x^2+7x+6}{x^2-1}$ and simplify for $x \neq -6$ or 1.

After factoring, this expression becomes: $\frac{x(x+6)}{(x+6)(x+6)} \cdot \frac{(x+1)(x+6)}{(x+1)(x-1)}$

After multiplying, reorder the factors: $\frac{(x+6)}{(x+6)} \cdot \frac{(x+6)}{(x+6)} \cdot \frac{x}{(x-1)} \cdot \frac{(x+1)}{(x+1)}$

Since $\frac{(x+6)}{(x+6)} = 1$ and $\frac{(x+1)}{(x+1)} = 1$, simplify: $1 \cdot 1 \cdot \frac{x}{(x-1)} \cdot 1 \Rightarrow \frac{x}{(x-1)}$

Example 4: Divide $\frac{x^2-4x-5}{x^2-4x+4} \div \frac{x^2-2x-15}{x^2+4x-12}$ and simplify for $x \neq 2, 5, -3,$ or -6.

First, change to a multiplication expression: $\frac{x^2-4x-5}{x^2-4x+4} \cdot \frac{x^2+4x-12}{x^2-2x-15}$

Then factor each expression: $\frac{(x-5)(x+1)}{(x-2)(x-2)} \cdot \frac{(x-2)(x+6)}{(x-5)(x+3)}$

After multiplying, reorder the factors: $\frac{(x-5)}{(x-5)} \cdot \frac{(x-2)}{(x-2)} \cdot \frac{(x+1)}{(x-2)} \cdot \frac{(x+6)}{(x+3)}$

Since $\frac{(x-5)}{(x-5)} = 1$ and $\frac{(x-2)}{(x-2)} = 1$, simplify to get: $\frac{(x+1)(x+6)}{(x-2)(x+3)} \Rightarrow \frac{x^2+7x+6}{x^2+x-6}$

Note: From this point forward in the course, you may assume that all values of x that would make a denominator zero are excluded.

Review & Preview

10-18. Now David wants to solve the equation $4000x - 8000 = 16{,}000$.

a. What easier equation could he solve instead that would give him the same solution? (In other words, what equivalent equation has easier numbers to work with?)

b. **Justify** that your equation in part (a) is equivalent to $4000x - 8000 = 16{,}000$ by showing that they have the same solution.

c. David's last equation to solve is $\frac{x}{100} + \frac{3}{100} = \frac{8}{100}$. Write and solve an equivalent equation with easier numbers that would give him the same answer.

10-19. Find the slope and y-intercept of each line below.

a. $y = -\frac{6}{5}x - 7$

b. $3x - 2y = 10$

c. The line that goes through the points (5, –2) and (8, 4).

10-20. Solve the systems of equations below using any method.

a. $3x - 3 = y$
$6x - 5y = 12$

b. $3x - 2y = 30$
$2x + 3y = -19$

10-21. Simplify the expressions below.

a. $\frac{x^2-8x+16}{3x^2-10x-8}$ for $x \neq -\frac{2}{3}$ or 4

b. $\frac{10x+25}{2x^2-x-15}$ for $x \neq -\frac{5}{2}$ or 3

c. $\frac{(k-4)(2k+1)}{5(2k+1)} \div \frac{(k-3)(k-4)}{10(k-3)}$ for $k \neq 3, 4$, or $-\frac{1}{2}$

10-22. Solve the equations below. Check your solution(s).

a. $\frac{m}{6} = \frac{m+1}{5}$

b. $\frac{3x-5}{2} = \frac{4x+1}{4}$

c. $\frac{8}{k} = \frac{14}{k+3}$

d. $\frac{x}{9} = 10$

10-23. A piece of metal at 20°C is warmed at a steady rate of 2 degrees per minute. At the same time, another piece of metal at 240°C is cooled at a steady rate of 3 degrees per minute. After how many minutes is the temperature of each piece of metal the same? Explain how you found your answer.