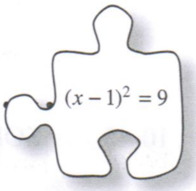
10.2.1 How can I solve it?



Multiple Methods for Solving Equations

So far in this course you have developed many different methods for solving equations, such as adding things to both sides of the equation or multiplying each term by a number to eliminate fractions. But how would you solve a complicated equation such as the one shown below?

$$(\sqrt{|x+5|}-6)^2+4=20$$

By looking at equations in different ways, you will be able to solve some equations much more quickly and easily. These new approaches will also allow you to solve new kinds of equations you have not studied before. As you solve equations in today's lesson, ask your teammates these questions:

How can you see it?

Is there another way?

10-44. DIFFERENT METHODS TO SOLVE AN EQUATION

By the end of this section you will be able to solve the equation $(\sqrt{|x+5|}-6)^2+4=20$. This equation is very complex and will require you to look at solving equations in new ways. To be prepared for other strange and unfamiliar equations, you will first examine all of the solving tools you currently have by solving a comparatively easier equation:

$$4(x+3)=20$$

Your Task: With your team, solve $4(x+3)=20$ for x in *at least* two different ways. Explain how you found x in each case and be prepared to share your explanations with the class.

Further Guidance

10-45. SOLVING BY REWRITING

David wants to find x in the equation $4(x+3)=20$. He said, "*I can rewrite this equation by distributing the 4 on the left-hand side.*" After distributing, what should his new equation be? Solve this equation using David's method.

10-46. SOLVING BY UNDOING

Juan says, "I see the whole thing a different way." Here is how he explains his approach to solving $4(x+3)=20$, which he calls "undoing": "*Instead of distributing first, I want to eliminate the 4 from the left side by undoing the multiplication.*"

a. What can Juan do to both sides of the equation to remove the 4? Why does this work?

b. Solve the equation using Juan's method. Did you get the same result as David?

c. Why is it appropriate for this method to be called "undoing"?

10-47. SOLVING BY LOOKING INSIDE

Kenya said, "I solved David's equation in a much quicker way!" She solved the equation $4(x+3)=20$ with an approach that she calls "looking inside." Here is how she described her thinking: "*I think about everything inside the parentheses as a group. After all, the parentheses group all that stuff together. I think the contents of the parentheses must be 5.*"

a. Why must the expression inside the parentheses equal 5?

b. Write an equation that states that the contents of the parentheses must equal 5. Then solve this equation. Did you get the same result as with David's method?

Further Guidance section ends here.

10-48. THE THREE METHODS

a. Find the Math Notes box for this lesson and read it with your team.

b. Match the names of approaches on the left with the examples on the right.

1. Rewriting
2. Looking inside
3. Undoing

i. "If $3+(4n-4)=12$, then $(4n-4)$ must equal 9…"

ii. "Subtracting is the opposite of adding, so for the equation $3(x-7)+4=23$, I can start by subtracting 4 from both sides…"

iii. "This problem might be easier if I turned $4(2x-3)$ into $8x-12$…"

10-49. For each equation below, decide whether it would be best to rewrite, look inside, or undo. Then solve the equation, showing your work and writing down the name of the approach you used. Check your solutions, if possible.

a. $\frac{2x-8}{10}=6$

b. $4+(x \div 3)=9$

c. $\sqrt{3x+3}=6$

d. $8-(2x+1)=3$

e. $\sqrt{x}+4=9$

f. $\frac{x}{3}-\frac{x}{9}=6$

10-50. Consider the equation $(x-7)^2=9$.

a. Solve this equation using *all three* approaches studied in this lesson. Make sure each team member solves the equation using all three approaches.

b. Did you get the same solution using all three approaches? If not, why not?

c. Of the three methods, which do you think was the most efficient method for this problem? Why?

MATH NOTES

METHODS AND MEANINGS

Methods to Solve One-Variable Equations

Here are three different approaches you can take to solve a one-variable equation:

Rewriting: Use algebraic techniques to rewrite the equation. This will often involve using the Distributive Property to get rid of parentheses. Then solve the equation using solution methods you know.

$$\begin{aligned} 5(x-1) &= 15 \\ 5x-5 &= 15 \\ 5x &= 20 \\ x &= 4 \end{aligned}$$

Looking inside: Choose a part of the equation that includes the variable and is grouped together by parentheses or another symbol. (Make sure it includes *all* occurrences of the variable!) Ask yourself, "What must this part of the equation equal to make the equation true?" Use that information to write and solve a new, simpler equation.

$$\begin{aligned} 5(x-1) &= 15 \\ 5(\;3\;) &= 15 \\ x-1 &= 3 \\ x &= 4 \end{aligned}$$

Undoing: Start by undoing the *last* operation that was done to the variable. This will give you a simpler equation, which you can solve either by undoing again or with some other approach.

$$\begin{aligned} \tfrac{5(x-1)}{5} &= \tfrac{15}{5} \\ x-1 &= 3 \\ +1 &= +1 \\ x &= 4 \end{aligned}$$

Review & Preview

10-51. Read the statements made by Hank and Frank below.

Hank says, "The absolute value of 5 is 5."

Frank says, "The absolute value of –5 is 5."

a. Is Hank correct? Is Frank correct?

b. How many different values for x make the equation $|x| = 5$ true?

10-52. Use the results from problem 10-51 to help you find all possible values for x in each of the following equations.

a. $|x| = 4$

b. $|x| = 100$

c. $|x| = -3$

d. $|x - 2| = 5$

10-53. Which of the expressions below are equal to 1? (Note: More than one answer is possible!)

a. $\frac{2x+3}{3+2x}$

b. $\frac{6x-12}{6(x-2)}$

c. $\frac{(2x-3)(x+2)}{2x^2+x-6}$

d. $\frac{x}{2} \div \frac{2}{x}$

10-54. Solve the inequalities below. Write each solution as an inequality.

a. $8 + 3x > 2$

b. $\frac{2}{3}x - 6 \le 2$

c. $-2x - 1 < -3$

d. $\frac{5}{x} \le \frac{1}{3}$

10-55. For the equation $\frac{3}{200} + \frac{x}{50} = \frac{7}{100}$:

a. Find a simpler equivalent equation (i.e., an equivalent equation with no fractions) and solve for x.

b. Which method listed in this lesson's Math Notes box did you use in part (a)?

10-56. Mr. Nguyen has decided to divide \$775 among his three daughters. If the oldest gets twice as much as the youngest, and the middle daughter gets \$35 more than the youngest, how much does each child get? Write an equation and solve it. Be sure to identify your variables.