10.2.2 How many solutions?

$(x-1)^2 = 9$

Determining the Number of Solutions

So far in this course you have seen many types of equations – some with no solution, some with one solution, others with two solutions, and still others with an infinite number of solutions! Is there any way to predict how many solutions an equation will have without solving it? Today you will focus on this question as you study quadratic equations written in perfect square form and equations with an absolute value. As you work with your team, ask the following questions:

Is there another way?

How do you see it?

Did you find all possible solutions?

10-57. The quadratic equation below is written in **perfect square form**. It is called this because the term $(x-3)^2$ forms a square when built with tiles. Solve this quadratic equation using one of the methods you studied in Lesson 10.2.1.

$$(x-3)^2 = 12$$

a. How many solutions did you find?

b. Write your answer in **exact** form. That is, write it in a form that is precise and does not have any rounded decimals.

c. Write your answer in **approximate** form. Round your answers to the nearest hundredth (0.01).

10-58. THE NUMBER OF SOLUTIONS

The equation in problem 10-57 had two solutions. However, from your prior experience you know that some quadratic equations have no solutions and some have only one solution. How can you quickly determine how many solutions a quadratic equation has?

With your team, solve the equations below. Express your answers in both **exact form** and **approximate form**. Look for patterns among those with no solution and those with only one solution. Be ready to report your patterns to the class.

a. $(x+4)^2 = 20$ b. $(7x-5)^2 = -2$ c. $(2x-3)^2 = 49$

d. $(5-10x)^2 = 0$ e. $(x+2)^2 = -10$ f. $(x+11)^2 + 5 = 5$

10-59. Use the patterns you found in problem 10-58 to determine quickly how many solutions each quadratic below has. You do not need to solve the equations.

a. $(5m-2)^2 + 6 = 0$ b. $(4+2n)^2 = 0$ c. $11 = (7+2x)^2$

10-60. Consider the equation $|2x-5|=9$.

a. How many solutions do you think this equation has? Why?

b. Which of the three solution approaches do you think will work best for this equation?

c. With your team, solve $|2x-5|=9$. Record your work carefully as you go. Check your solution(s).

10-61. The equation $|2x-5|=9$ from problem 10-60 had two solutions. Do you think all absolute-value equations must have two solutions? Consider this as you answer the questions below.

a. Can an absolute-value equation have no solution? With your team, create an absolute-value equation that has no solution. How can you be sure there is no solution?

b. Likewise, create an equation with an absolute value that will have only one solution. **Justify** why it will have only one solution.

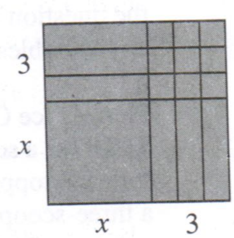
10-62. Is there a **connection** between how to determine the number of solutions of a quadratic in perfect square form and how to determine the number of solutions of an equation with an absolute value? In your Learning Log, describe this connection and explain how you can determine how many solutions both types of equations have. Be sure to include examples for each. Title this entry "Number of Solutions" and include today's date.

MATH NOTES

METHODS AND MEANINGS

Perfect Square Form of a Quadratic

When a quadratic equation is written in the form $(x-a)^2=b^2$, such as the one below, we say it is in **perfect square form**. Notice that when the quadratic expression on the left side of the equation below is built with tiles, it forms a square, as shown at right.



$$(x+3)^2=25$$

Review & Preview

10-63. Solve these equations, if possible. Each time, be sure you have found all possible solutions. Check your work and write down the name of the method(s) you used.

a. $(x+4)^2 = 49$

b. $3\sqrt{x+2} = 12$

c. $\frac{2}{x} + \frac{3}{10} = \frac{13}{10}$

d. $5(2x-1) - 2 = 13$

10-64. Is $x = -4$ a solution to $\frac{1}{3}(2x+5) > -1$? Explain how you know.

10-65. Multiply or divide the rational expressions below. Leave your answer in simplified form.

a. $\frac{(x+4)(2x-1)(x-7)}{(x+8)(2x-1)(3x-4)} \div \frac{(4x-3)(x-7)}{(x+8)(3x-4)}$

b. $\frac{2m^2+7m-15}{m^2-16} \cdot \frac{m^2-6m+8}{2m^2-7m+6}$

10-66. An **exponent** is shorthand for repeated multiplication. For example, $x^3 = x \cdot x \cdot x$. Use the meaning of an exponent to rewrite each of the expressions below.

a. $(3x-1)^2$

b. 7^4

c. m^3

d. w^{10}

10-67. Factor each of the following expressions completely. Be sure to look for any common factors.

a. $4x^2 - 12x$

b. $3y^2 + 6y + 3$

c. $2m^2 + 7m + 3$

d. $3x^2 + 4x - 4$

10-68. Write and solve an equation to answer the question below. Remember to define any variables you use.

Pierre's Ice Cream Shoppe charges \$1.19 for a scoop of ice cream and \$0.49 for each topping. Gordon paid \$4.55 for a three-scoop sundae. How many toppings did he get?