

**MAIN IDEA**

Find the measures of variation of a set of data.

**New Vocabulary**

measures of variation  
quartiles  
lower quartile  
upper quartile  
interquartile range  
outlier

**Math Online**

[glencoe.com](http://glencoe.com)

- Extra Examples
- Personal Tutor
- Self-Check Quiz

**GET READY for the Lesson**

**TEEN SPENDING** The average amount of money teens spend each week is given in the table.

1. Find the median of the data.
2. Organize the data into two groups: the top half and the bottom half. How many data values are in each group?
3. What is the median of each group?
4. Find the difference between the two numbers from Question 3.
5. Find the range of the data.
6. What does the range tell you about the amount of money spent each week for those ten countries?



**Top Ten Countries Average Weekly Teen Spending**

Norway	\$49.70
Sweden	\$41.70
Brazil	\$41.30
Argentina	\$40.50
Hong Kong	\$38.00
United States	\$37.60
Denmark	\$37.40
Singapore	\$34.10
Greece	\$32.90
France	\$31.30

Source: Media Wiley

**Measures of variation** are used to describe the distribution of the data. In Lesson 11-4, you used the range to describe how “spread out” the data are. The range is one measure of variation. **Quartiles** are values that divide a set of data into four equal parts. Recall that the median separates a set of data in two equal parts.

lower half					median	upper half				
\$31.30	\$32.90	\$34.10	\$37.40	\$37.60	↓	\$38.00	\$40.50	\$41.30	\$41.70	\$49.70

The median of the lower half of a set of data is the **lower quartile** or **LQ**.

The median of the upper half of a set of data is the **upper quartile** or **UQ**.

So, one half of the data lie between the lower quartile and the upper quartile. Another measure of variation is the **interquartile range**.

**Interquartile Range****Key Concept**

The interquartile range is the range of the middle half of the data. It is the difference between the upper quartile and the lower quartile.



# EXAMPLE

## Find Measures of Variation

- ① **MOVIES** Find the measures of variation for the data in the table.

Range 204 - 20 or 184 films

Median, Upper Quartile, and Lower Quartile

Order the numbers from least to greatest.

lower quartile			median		upper quartile		
20	24	41	85	85	123	139	204
$\frac{24 + 41}{2} = 32.5$			$\frac{85 + 85}{2} = 85$		$\frac{123 + 139}{2} = 131$		

The median is 85, the lower quartile is 32.5, and the upper quartile is 131.

Interquartile Range upper quartile - lower quartile  
= 131 - 32.5 or 98.5

Themes of Sports Films	
Sport	Films
Auto Racing	85
Baseball	85
Basketball	41
Boxing	204
Football	123
Golf	24
Horse Racing	139
Wrestling	20

Source: Top Ten of Everything

### Study Tip

#### Value of Interquartile Range

A small interquartile range means that the data in the middle of the set are close together. A large interquartile range means that the data in the middle are spread out.

### CHECK Your Progress

- a. **ENTERTAINMENT** Determine the measures of variation for the data in the table.

#### DVD Prices in Dollars at Various Stores

14.95	19.99	24.99	17.99
14.99	14.95	23.49	15.89
15.99	21.95	17.99	15.99

order #'s

14.95 - min

14.95 14.99 + 15.89

14.99  $Q_1 = 15.44$

15.89

15.99

15.99

17.99

17.99

19.99 19.99 + 21.95

21.95  $Q_3 = 20.97$

23.49

24.99 - max

5 point summary

min = 14.95

$Q_1 = 15.44$

med = 16.99

$Q_3 = 20.97$

max = 24.99

Data that are more than 1.5 times the value of the interquartile range beyond either quartile are called **outliers**. An outlier is a data value that is either much *larger* or much *smaller* than the median.

# EXAMPLE

## Find Outliers

- ② **WINDS** Find any outliers for the data in the table.

Find the interquartile range.

$$12.4 - 9.0 = 3.4$$

Multiply the interquartile range by 1.5.

$$3.4 \times 1.5 = 5.1$$

Now subtract 5.1 from the lower quartile and add 5.1 to the upper quartile.

$$9.0 - 5.1 = 3.9$$

$$12.4 + 5.1 = 17.5$$

The only outlier is 35.1 because it is greater than 17.5.

#### Average Speeds of Winds

Station	Speed (mph)
Mt. Washington, NH	35.1
Boston, MA	12.4
Buffalo, NY	11.8
Detroit, MI	10.2
Lexington, KY	9.1
Pittsburgh, PA	9.0
Phoenix, AZ	6.2

Source: The World Almanac

### CHECK Your Progress

- b. **BUILDINGS** Find any outliers for the data in the table.

#### Tallest Buildings (ft), Houston, Texas

1,002	972	901	780	748	762
725	714	691	685	741	732

685 691 714 725 732 741 748 762 780 901 972 1002

$Q_1 = 719.5$

744.5  
median

$Q_3 = 840.5$

$$IQR = Q_3 - Q_1 \Rightarrow 840.5 - 719.5 = 121$$

$$719.5 - 181.5 = 538$$

no # over 1022 or less than 538



**EXAMPLE****Use Measures of Variation to Describe Data**

- 3 **SLEEP** Use the measures of variation to describe the data in the table at the right.

Find the measures of variation.

The range is  $19.9 - 1.9$ , or 18.

The median is 11.25.

The upper quartile is 17.05.

The lower quartile is 4.55.

The interquartile range is  $17.05 - 4.55$ , or 12.5.

**Number of Hours of Sleep for Selected Animals**

Brown Bat	19.9
Giant Armadillo	18.1
Infant Human	16.0
Cat	12.1
Bottle-Nosed Dolphin	10.4
Gray Seal	6.2
Horse	2.9
Giraffe	1.9

Source: Neuroscience for Kids

The spread of the data is 18 hours. The median is 11.25 hours. One fourth of the animals got at or below 4.55 hours of sleep and one fourth of the animals got at or above 17.05 hours of sleep. The number of hours of sleep for half of the animals was in the interval 4.55–17.05.

**CHECK Your Progress**

- c. **CYCLING** Use the measures of variation to describe the data in the table at the right.

The spread of data is 28.5  
(36-8) The median is 9  
with upper  $Q=18$   
and lower  $Q=8$   
The IQR = 10  
and there are  
no outliers

8 8 9 18 36  
↓ med ↓  
 $Q_1 = 8$   $Q_3 = 18$   
 $IQR = 18 - 8 = 10$   
 $10 \times 1.5 = 15$   
 $8 - 15 = -7$   
 $18 + 15 = 33$   
 $-7 < 33$  > no outliers

**Number of Tour de France Wins**

France	36
Belgium	18
Italy	9
Spain	8
United States	8

Source: World Almanac for Kids

**CHECK Your Understanding**

**LANGUAGE** For Exercises 1–5, use the data in the table on the right.

Example 1  
(p. 600)

- Determine the range of the data.
- Find the median and the upper and lower quartiles.
- What is the interquartile range of the data?

Example 2  
(p. 600)

- Identify any outliers.

Example 3  
(p. 601)

- Use the measures of variation to describe the data in the table.

**U.S. Non-English Language Spoken at Home**

Language	Speakers (millions)
Spanish	28.1
Chinese	2.0
French	1.6
German	1.4
Tagalog	1.2
Vietnamese	1.0
Italian	1.0
Korean	0.9

Source: U.S. Census Bureau



# Practice and Problem Solving

## HOMEWORK HELP

For Exercises	See Examples
6, 7, 10, 11, 14, 15, 18, 19	1
8, 12, 16, 20	2
9, 13, 17, 21	3

**SYRUP** For Exercises 6–9, use the data in the table at the right.

- What is the range of the data?
- Find the median, the upper and lower quartiles, and the interquartile range of the data.
- Identify any outliers.
- Use the measures of variation to describe the data in the table.

**EXERCISE** For Exercises 10–13, use the data in the table at the right.

- What is the range of the data?
- Find the median, upper and lower quartiles, and the interquartile range of the data.
- Identify any outliers.
- Use the measures of variation to describe the data in the table.

**SPACE** For Exercises 14–17, use the data in the table at the right.

- What is the range of the data?
- Find the median, upper and lower quartiles, and the interquartile range for the data.
- Identify any outliers.
- Use the measures of variation to describe the data in the table.

**ANIMALS** For Exercises 18–21, use the data in the table at the right.

- What is the range of the data?
- Find the median, upper and lower quartiles, and the interquartile range for the data.
- Identify any outliers.
- Use the measures of variation to describe the data in the table.

- GOLF** Brandon's golf scores relative to par were  $-1, -2, 4, -6, 3, -1$ , and  $-3$ . Rashan's scores were  $-5, 5, 0, 4, -1, -4$ , and  $-3$ . Find the measures of variation of both person's scores. Then describe any similarities or differences in the measures of variation.

**Annual Production of Maple Syrup (gallons)**

Vermont	430,000
Maine	265,000
New York	210,000
Wisconsin	76,000
Michigan	59,000

Source: World Almanac for Kids

**Calories Burned per Minute of Exercise**

Jogging (6 mph)	8
Jumping Rope	7
Basketball	7
Soccer	6
Bicycling (9.4 mph)	5
Downhill Skiing	5
Walking (4 mph)	4

Source: World Almanac for Kids

**Number of U.S. Shuttle Launches 1981–2005**

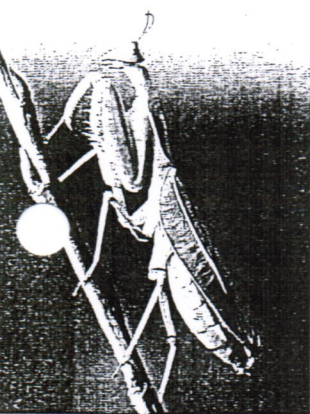
1981–1985	23
1986–1990	15
1991–1995	28
1996–2000	28
2001–2005	13

Source: NASA

**Number of Species in the Animal Kingdom**

Arthropods	1,100,000
Fish	24,500
Birds	9,000
Mammals	9,000
Reptiles	8,000
Amphibians	5,000

Source: World Almanac for Kids



**Real-World Link . . .**  
Crustaceans, insects, and spiders are all arthropods. There are about 750,000 species of insects alone.

Source: World Almanac for Kids



**RIVERS** For Exercises 23–27, use the table at the right.

23. Which continent has a greater range of length of rivers?
24. Find the measures of variation for each continent.
25. Compare the modes and the interquartile ranges of the length of rivers.
26. Select the appropriate measure of central tendency or range to describe the length of rivers for each continent. Justify your response.
27. Describe the length of rivers of Africa and South America, using both the measures of central tendency and variation.

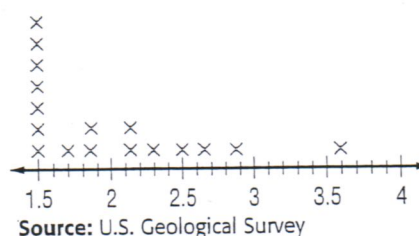
Length (miles) of Principal Rivers			
Africa		South America	
4,160	700	4,000	1,300
2,900	660	2,485	1,100
2,590	500	2,100	1,000
1,700		2,013	1,000
1,300		1,988	1,000
1,100		1,750	956
1,100		1,677	910
1,020		1,600	808
1,000		1,584	400
1,000		1,400	150

Source: *The World Almanac*

**EARTHQUAKES** For Exercises 28–30, use the line plot.

28. Find the range, mean, median, mode, upper and lower quartiles, and the interquartile range for the data.
29. Identify any outliers.
30. Use the measures of variation to describe the data in the table.

Magnitude of Earthquakes in the Central U.S. for September 2005



Source: U.S. Geological Survey

**BRIDGES** For Exercises 31–33, use the table at the right.

31. Find the length of the Golden Gate Bridge if the median is 4,000 feet.
32. Find the length of the Akashi Kaikyo Bridge if the range is 2,566 feet.
33. The 11th longest suspension bridge in the world is the Tagus River Bridge in Portugal, with a length of 3,323 feet. Describe how the measures of variation are affected if this data value is included.

10 Longest Suspension Bridges in the World		
Bridge	Country	Length (ft)
Akashi Kaikyo	Japan	y
Great Belt Link	Denmark	5,328
Humber River	England	4,626
Verrazano Narrows	United States	4,260
Golden Gate	United States	x
Mackinac Straits	United States	3,800
Minami Bian-Seto	Japan	3,668
Second Bosphorous	Turkey	3,576
First Bosphorous	Turkey	3,524
George Washington	United States	3,500

Source: *Structural Steel Designer's Handbook*

34. **FIND THE DATA** Refer to the Data File on pages 16–19. Choose some data and write a real-world problem in which you would determine measures of central tendency and measures of variation.

**Real-World Link . . . .**  
The Verrazano Narrows Bridge in New York City is the longest bridge in the United States. Each of its two towers weighs 27,000 tons.  
Source: Metropolitan Transit Authority

**EXTRA PRACTICE**  
See pages 697, 710.

**H.O.T. Problems**

35. **OPEN ENDED** Create a list of data with at least eight numbers that has an interquartile range of 20 and one outlier.
36. **CHALLENGE** Create two different sets of data that have the same range but different interquartile ranges. Then create two different sets of data that have the same median and same quartiles, but different ranges.
37. **WRITING IN MATH** Explain why the interquartile range is not affected by very high or low values in the data.

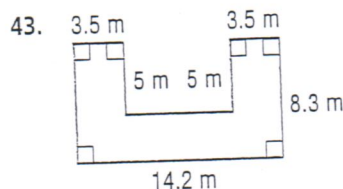
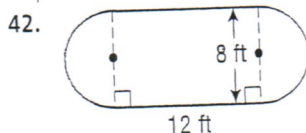
**TEST PRACTICE**

38. Which of the following statements is never true concerning the measures of variation of a set of data?
- A Half the data lie within the interquartile range.
  - B Three fourths of the data lie above the lower quartile.
  - C The median, the lower quartile, and the upper quartile separate the data into three equal parts.
  - D 50% of the data lie below the median.
39. The number of Grand Slam singles titles won by twelve tennis players are 14, 8, 7, 6, 5, 5, 10, 11, 8, 8, 6, and 7. Which of the following statements is *not* supported by these data?
- F Half of the titles won are below 7.5 and half are above 7.5.
  - G The spread of the data is 9 titles.
  - H An outlier of the data is 11 titles.
  - J About one fourth of the titles won are at or above 9 titles.

**Spiral Review**

40. **HEIGHTS** The heights, in inches, of the Allen family are 72, 68, 48, 71, and 67. Find the mean, median, mode, and range. Round to the nearest tenth if necessary. (Lesson 11-4)
41. **NATIONAL PARKS** Wyoming has 3,159 square miles of Yellowstone National Park while Montana has 264 square miles and Idaho has 49 square miles. Construct a circle graph to show what part of Yellowstone National Park is in each state. (Lesson 11-3)

**MEASUREMENT** Find the area of each figure. Round to the nearest tenth. (Lesson 7-2)

**▶ GET READY for the Next Lesson**

**PREREQUISITE SKILL** Graph each set of points on a number line. (Lesson 1-3)

44. {3, 5, 8, 9, 10}      45. {13, 15, 20, 27, 31}      46. {9, 13, 16, 17, 21}      47. {3, 9, 10, 15, 19}