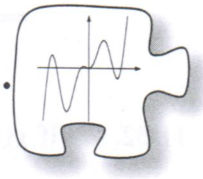
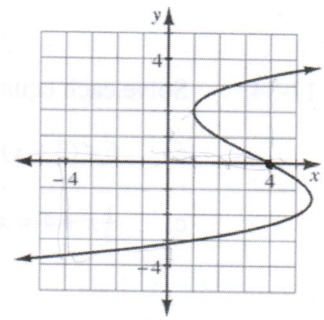
11.1.4 What can go in? What can come out?



Domain and Range

So far you have described relations using intercepts and symmetry. You also have noticed that sometimes relations are functions. Today you will finish your focus on relations by describing the inputs and outputs of relations.

11-38. Examine the graph of the relation $h(x)$ at right. Use it to estimate:



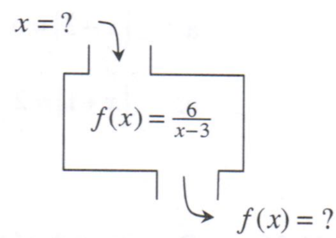
a. $h(4)$

b. $h(1)$

c. $h(-4)$

d. Is this relation a function? Why or why not?

11-39. Examine the relation shown at right.



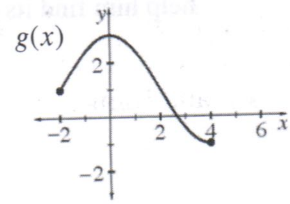
a. Find $f(-3)$, $f(0)$, and $f(2)$.

b. Find $f(3)$. What happened?

c. Are there any other numbers that cannot be evaluated by this relation? In other words, are there any other values that cannot be x? Explain how you know.

d. The set (collection) of numbers that can be used for x in a relation is called the **domain** of the relation. The domain is a description or list of all the possible x-values for the relation. Describe the domain of $f(x) = \frac{6}{x-3}$.

11-40. Now examine $g(x)$ graphed at right.



a. Is $g(x)$ a function? How can you tell?

b. Which x-values have points on the graph? That is, what is the domain of $g(x)$?

c. What are the possible outputs for $g(x)$? This is called the **range** of the relation.

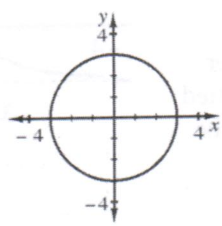
d. Ricky thinks the range of $g(x)$ is: −1, 0, 1, 2, and 3. Is he correct? Why or why not?

11-41. FINDING DOMAIN AND RANGE

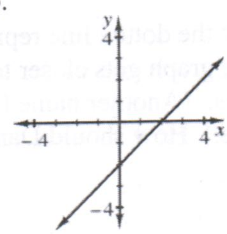
The domain and range are good descriptors of a relation because they help you know what numbers can go into and come out of a relation. The domain and range can also help you set up useful axes when graphing and help you describe special points on a graph (such as a missing point or the lowest point).

Work with your team to describe in words the domain and range of each relation below.

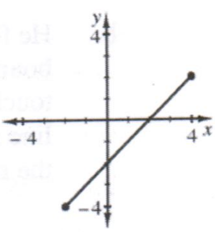
a.



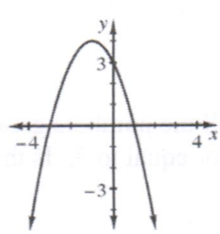
b.



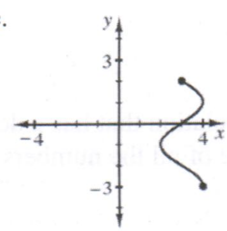
c.



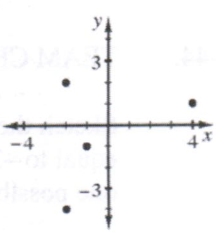
d.



e.



f.



11-42. Chiu loves tables! He has decided to make the table below for a relation $f(x)$ to help him find its domain and range.

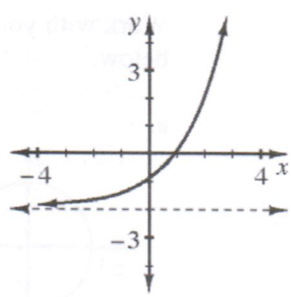
x	-3	-2	-1	0	1	2	3
$f(x)$	5	0	-3	-4	-3	0	5

a. From his table, can you tell what the domain of $f(x)$ is? Why or why not?

b. From the table, can you tell the range of $f(x)$? Why or why not?

c. Is using a table an effective way to determine the domain and range of a relation?

11-43. Daniel is thinking about the relation shown at right.



a. He noticed that the curve continues to the left and to the right. What is the domain of this relation?

b. He found out that the dotted line represents a boundary that the graph gets closer to but never touches or crosses. (Another name for this dotted line is **asymptote**.) How should Daniel describe the range?

11-44. TEAM CHALLENGE

Sketch the graph of a relation that has a domain of all the numbers greater than or equal to –2 and a range of all the numbers less than or equal to 3. Is there more than one possible answer?

MATH NOTES

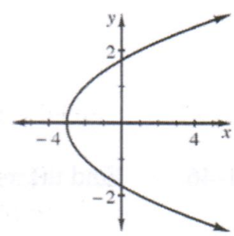
METHODS AND MEANINGS

Relations and Functions

A **relation** establishes a correspondence between its inputs and outputs (in math language called "sets"). For equations, it establishes the relationship between two variables and determines one variable when given the other. Some examples of relations are:

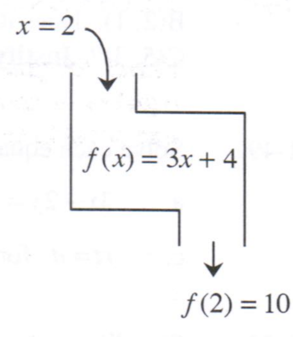
$$y = x^2,\ y = \frac{x}{x+3},\ y = -2x + 5$$

Since the value of y usually depends on x, y is often referred to as the **dependent variable**, while x is called the **independent variable**.



The set of possible inputs of a relation is called the **domain**, while the set of all possible outputs of a relation is called the **range**. For example, notice that all the points on the graph at right have x-values that are greater than or equal to –3. The arrows on the graph indicate that the graph will continue to expand to the right. Thus, the entire domain is the set of numbers that are greater than or equal to –3. Likewise, since each y-value has a corresponding point on the graph, then the range is the set of all numbers. This is also referred to as the set of **all real numbers**. In the future, this course will refer to these as "all numbers."

A **relation** is called a **function** if there exists no more than one output for each input. If a relation has two or more outputs for a single input value, it is not a function. For example, the relation graphed above is not a function because there are two y-values for each x-value greater than –3.



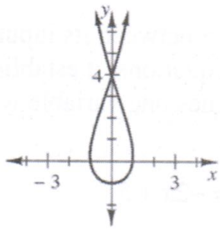
Functions are often given names, most commonly "f," "g," or "h." The notation $f(x)$ represents the output of a function, named "f" when x is the input. It is read "f of x." The notation $f(2)$, read "f of 2," represents the output of function f when $x = 2$. In the example at right, $f(2) = 10$.

The equations $y = 3x + 4$ and $f(x) = 3x + 4$ represent the same function. Notice that this notation is interchangeable; that is, $y = f(x)$.

Review & Preview

11-45. Which of the relations below are functions? If a relation is not a function, give a reason to support your conclusion.

a.



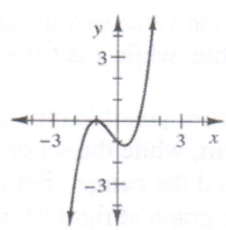
b.

x	y
–3	19
5	19
19	0
0	–3

c.

x	7	–2	0	7	4
y	10	0	10	3	0

d.



11-46. Find the x- and y-intercepts for the relations in problem 11-45.

11-47. Marisol and Mimi walked the same distance from their school to a shopping mall. Marisol walked 2 miles per hour, while Mimi left 1 hour later and walked 3 miles per hour. If they reached the mall at the same time, how far from the mall is their school?

11-48. A line passes through the points A(–3, –2) and B(2, 1). Does it also pass through the point C(5, 3)? **Justify** your conclusion.

11-49. Solve each equation below for the indicated variable.

a. $3x - 2y = 18$ for x

b. $3x - 2y = 18$ for y

c. $rt = d$ for r

d. $C = 2\pi r$ for r

11-50. Simplify each expression below.

a. $\frac{3x^2+8x+5}{x^2-5x-6} \cdot \frac{2x-5}{3x+5}$

b. $\frac{x^2+x-12}{x^2-x-6} \div \frac{x-5}{x^2-3x-10}$