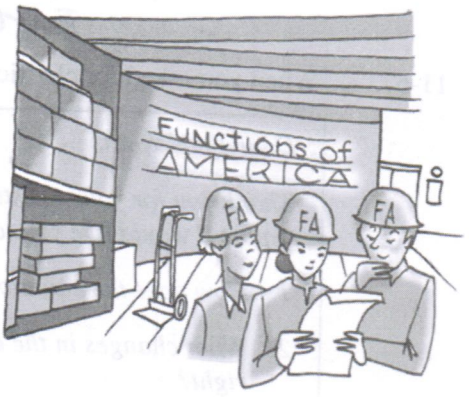
11.1.6 How does it change?

Transformation of a Function

11-66. PROMOTION OPPORTUNITY

You and your co-workers at Functions of America have received the following note from your boss. Read the letter and complete the task that follows.



To My Faithful Employees,

I have been pleased to read in the "Relations Daily" about the high customer-service satisfaction ratings of this company. Now I want to expand to control the function-rental industry.

Before Functions of America can begin renting out our expensive graphs, I need an equation attached to each one. That way, when a graph is returned to the company, employees will be able to verify that it is the same function that was originally rented to the customer. Also, if the graph of the function was damaged or switched for a less sophisticated graph, we will then be able to prosecute the customer to the full extent of the law.

Employees will be given a designated time to explore their new and innovative function with a partner. Please be ready to report on how the numbers in a rule change its graph.

At the end of the day, every employee will be challenged to identify the equation of a function correctly by observing only its graph. Doing so will earn you a management position. I wish all of you the best of luck.

Sincerely,
Freda Function, CEO, Functions of America

Your Task: Your teacher will assign your team one of the functions below. Explore the graph of your function as a, h, and k change values. Choose positive, negative, and zero values for a, h, and k to uncover all possible patterns. Reflect on the relationships you find between the graph and its equation. Discuss your observations with your study team and record your results on paper.

(1) $f(x)=a(x-h)^3+k$ (2) $f(x)=a(x-h)^2+k$ (3) $f(x)=a\sqrt{x-h}+k$

(4) $f(x)=a|x-h|+k$ (5) $f(x)=\frac{a}{x-h}+k$ (6) $f(x)=2^{a(x-h)}+k$

Problem continues on next page →

11-66. *Problem continued from previous page.*

Discussion Points

What is the goal of this investigation?

What is the best way to choose values of a, h, and k to see a pattern?

Further Guidance

11-67. When you asked for clarification, your boss sent you the following note:

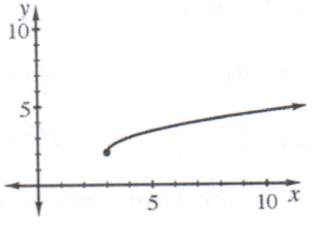
> *Dear Employees,*
>
> *Thank you for your questions. I am sorry I was so vague. In your report, I would like you to tell me:*
>
> 1. *How does the equation affect how "skinny" or "wide" the graph is?*
> 2. *What changes in the equation move the graph up or down? Left or right?*
> 3. *Is there a way to change the equation so that the function turns "upside down"?*
>
> *Use your graphing technology to test different values of a, h, and k to discover the answers to the questions above. Examine only one letter at a time so that you can find patterns quickly. For example, if you want to see what the value of a does to the graph of a function, then change a while you keep h and k the same.*
>
> *Good luck!*
> *Ms. Function*

Further Guidance section ends here.

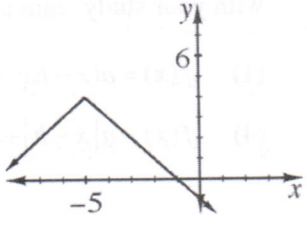
11-68. PROMOTION CHALLENGE

Here is your opportunity to impress your boss. Find the equation for each relation graphed below. Remember the observations you made in problem 11-66 and pay close attention to details.

a.



b.



11-69. EXTENSION

How do the domain and range of a function change when it moves? To answer this question, examine what happens as the square-root function $f(x)=\sqrt{x}$ is moved ("translated").

a. Describe the domain and range of $f(x)=\sqrt{x}$.

b. Now describe the domain and range of $g(x)=\sqrt{x+2}-3$.

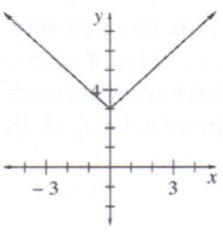
c. Are the domain and range for $f(x)$ and $g(x)$ above the same? If not, how are they different?

Review & Preview

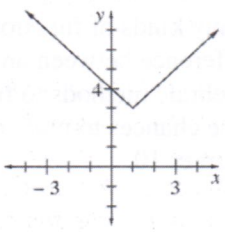
11-70. Match each rule below with its corresponding graph. Can you do this without making any tables?

a. $y=|x-1|$ b. $y=|x|+3$ c. $y=|x-1|+3$

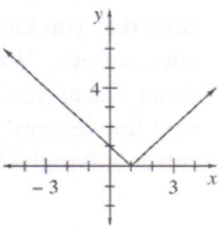
1)



2)



3)



11-71. Graph the rule $y=-x^2+4x$ and label its intercepts and vertex.

11-72. If $f(x)=7+|x|$ and $g(x)=x^3-5$, then find:

a. $f(-5)$ b. $g(4)$ c. $f(0)$

d. $f(2)$ e. $g(-2)$ f. $g(0)$

11-73. Solve for x in each equation below.

a. $2x=8$ b. $2x+2=10$

c. $6x+2-4x=10$ d. $2(3x+1)-4x=10$

e. Check your solutions for the equations above. What do you notice?

11-74. Multiply each expression below using generic rectangles.

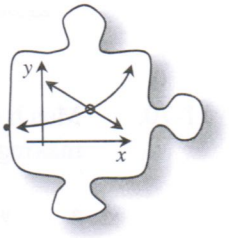
a. $(4x-1)(3x+2)$ b. $(m+1)(3m-2)$ c. $(k-4)(6-5k)$

11-75. Solve the following inequalities for x. Graph your solutions on a number line.

a. $3x-5 \le 7+2x$ b. $|x|-3<7$

c. $5(2-x)+6>16$ d. $|x+2|>3$

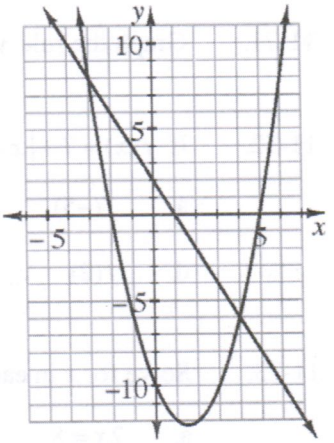
11.2.1 Intercept or intersect?



Intercepts and Intersections

Now that you know about many kinds of functions, you will look more closely at intercepts and intersections. What is the difference between an intercept and a point of intersection? Think about this as you develop algebraic methods to find points where two functions cross. In the next few lessons you will have chances to practice your quadratic-solving skills as well as your newer solving skills from Chapter 10.

11-76. Examine the graphs of the parabola $y = x^2 - 3x - 10$ and the line $y = -2x + 2$ at right.



a. Name all x- and y-intercepts for the parabola.

b. Name all x- and y-intercepts for the line.

c. Where do the graphs intersect each other?

d. The words "intersect" and "intercept" look and sound a lot alike, but what do they mean? How are they alike? How are they different?